The Morphological Top-Hat Operator Generalised to Multi-channel Images

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Abstract

The morphological top-hat operator for greyscale images is part of the basic toolbox of mathematical morphology operators. We discuss two ways of generalising the top-hat operator to multi-channel images, such as colour images. The first method presented is the use of a vectorial order in the relevant vector space. The second is based on the demonstration that the top-hat operator can be rewritten in terms of increments. These increments can be replaced by any vectorial distance function, removing the requirement to first impose an order on the vectors. We present examples of the use of the suggested top-hat operators in feature detection in colour images and defect detection in texture.

1. Introduction

The morphological top-hat operator [5] for greyscale images is part of the basic toolbox of mathematical morphology operators [7]. Its function is to detect contrasted objects on non-uniform backgrounds. For greyscale images, there are two versions: the white top-hat $\rho(f)$ is the difference between an image f and its opening $\gamma(f)$, i.e. $\rho(f) = f - \gamma(f)$, which extracts bright structures. The black top-hat $\rho^*(f) = \varphi(f) - f$, the difference between image f and its closing, extracts dark structures.

For multi-channel (or vector-valued) images, i.e. images of the form $f(x) = \mathbf{v}$ which contain a vector $\mathbf{v}$ at each grid position x, there exist a number of ways in which the top-hat operator can be applied. The most obvious is to extract one of the channels or a combination of channels in which the objects to be extracted are clearly visible and apply one of the standard greyscale top-hat operators, as done, for example, in [1]. The disadvantage of this approach is that one loses all multi-channel information, which for some applications could be useful to take into account when calculating the difference. Such an application could be the use of the top-hat operator in the CIELAB colour space, in which one could take advantage of the metric defined in the space to calculate accurate colour differences. Keeping all the channels in the result of the opening or closing operator can be done by using a vectorial order with the morphological operator. The subtraction in the standard top-hat is then replaced by a pixelwise vectorial distance. This approach is described in Section 2. The second approach which we introduce is to rewrite the top-hat in terms of increments, which obviates the requirement to choose a vectorial order. This approach is described in Section 3, and illustrated with examples from colour and texture analysis.

*This work is supported by the EU MUSCLE NoE FP6-507752.

2. Opening or closing-based top-hat

The most obvious way of extending the greyscale top-hat operator to multi-channel images is to use the standard formulation, but to use a vectorial order for the opening and closing. Many vectorial orders have been defined, such as the marginal, reduced and lexicographical orders [2]. In practice, use of the marginal order implies applying greyscale morphological operators to each channel independently and then combining the results to form a new vector-valued image. This has the disadvantage of possibly introducing new vectors (i.e. vectors which were not present in the initial image) into the resultant image. For colour images, this produces false colours [8]. In the case of the top-hat, if one carries out a complete top-hat operation on each channel, the resulting vectorial values are difficult to interpret. A top-hat operator which produces a greyscale result is to be preferred.

The lexicographical and reduced orders process the vectors as complete units, so that no new vectors appear in the result. The reduced order uses a function $g(\mathbf{v})$ (defined by the user) to calculate a scalar value for each vector, and the vectors are ordered based on this value. If g is not injective, then the vector order is not total, which means that pairs of vectors exist for which an order cannot be determined. To guarantee a total order, one can choose g to be injective or use a lexicographical order. These two orders can also be combined by using a non-injective g at the top-level of a lexicographical order, thereby creating a total order.

Proceedings of the 17th International Conference on Pattern Recognition (ICPR'04)
1051-4651/04 $ 20.00 IEEE

If one knows that the information of interest is in one of the channels, then one can use a lexicographical order with that channel placed at the top level. For example, for a colour image represented in a 3D-polar coordinate system (hue H, saturation S, brightness Y) for which the information of interest is in the saturation channel, the following lexicographical order can be used[1]:

$$\mathbf{v}_i > \mathbf{v}_j \text{ if } \begin{cases} S_i > S_j \\ \text{or} \\ S_i = S_j \text{ and } Y_i > Y_j \\ \text{or} \\ S_i = S_j \text{ and } Y_i = Y_j \\ \qquad \text{and } \angle\left(H_i, H_0\right) < \angle\left(H_j, H_0\right) \end{cases} \tag{1}$$

where $\mathbf{v}_i$ and $\mathbf{v}_j$ are vectors of colour coordinates, H_0 is an arbitrarily chosen hue origin, and the notation $\angle\left(\theta_i, \theta_j\right)$ indicates the acute angle between angles θ_i and θ_j. The top-hat is then determined by calculating a pixel-wise vectorial distance between the pixels in the initial image and in the opened or closed image, producing a greyscale image.

An example of a top-hat using this lexicographical order is shown in Figure 1. The aim is to extract the lines between the mosaic tiles of Figure 1a. In the luminance channel, these lines have values both above and below those of the tiles, but in the saturation channel shown in Figure 1b, the greyvalues of the lines are mostly below those of the tiles. A closing of this image using the lexicographical order of Equation 1 covers over the lines, and a pixelwise Euclidean distance between the pixels in the original colour image and the closed image produces Figure 1d.

3. Difference-based top-hat

In [4] it is shown that the top-hat can be written in terms of *increments* of pixel values as follows. Firstly, we recall that an opening $\gamma_B(x)$ by structuring element B at point x can be equivalently written as

$$\gamma_B(x) = \sup\left\{\inf\left[f(y), y \in B_i\right], i \in I\right\} \tag{2}$$

where $\{B_i, i \in I\}$ is the family of structuring elements which contain point x. Using this expression, the top-hat expression $f(x) - \gamma_B(x)$ can be rewritten in terms of increments of f as

$$-\sup\left\{\inf\left[f(y) - f(x), y \in B_i\right], i \in I\right\} \tag{3}$$

In [4] the next step was to replace the increment $[f(y) - f(x)]$ by an angular distance, namely the acute angle between any two angles. Here we make the observation that this increment can be replaced by any measure

[1]Note that one should take care that the normalisation by the brightness has been removed from the saturation channel, otherwise this saturation order is essentially useless.

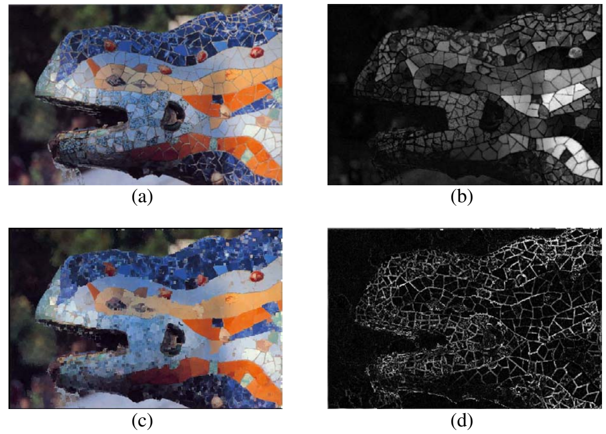

Figure 1. (a) Lizard image (size 544x360 pixels). (b) Saturation of image (a). (c) Morphological closing of image (a) using a lexicographical order with saturation at the top level. (d) Top-hat — the Euclidean distance between images (a) and (c).

of distance $D[f(x), f(y)]$ between two pixel values, for example the Euclidean distance between two RGB vectors. It is nevertheless important to notice that we are replacing the expression $[f(y) - f(x), y \in B_i] \in (-\infty, \infty)$ by the expression $\{D[f(x), f(y)], y \in B_i\} \in [0, \infty)$. Hence, if the structuring element origin is part of the structuring element, the expression $\inf\{D[f(x), f(y)], y \in B_i\}$ is always equal to zero. To prevent the result of this operator being a zero-valued image, one must rewrite it in the dual form which is written only in terms of suprema, as follows

$$\mathrm{TH}(x) = -\sup\left\{-\sup\left[D\left(f(x), f(y)\right), y \in B_i\right], i \in I\right\} \tag{4}$$

This expression is applied to every pixel x of an image to calculate the top-hat of the image.

One has a large choice of distance functions to use for $D[f(x), f(y)]$. The most obvious is the Euclidean distance in a colour space. If this is done in a properly calibrated CIELAB image, then the values obtained in the top-hat image can be interpreted as colour differences. As an example, such a top-hat using the Euclidean distance in the RGB space is applied to the colour image in Figure 2a, with the results shown in Figure 2d, in which the majority of the lines between the mosaic tiles have been extracted. For comparison, a standard top-hat applied to the luminance (Figure 2b) is also shown. As some of the lines between the tiles have lower luminance and some higher luminance than the tiles, it is necessary to take the supremum of the images produced by the black top-hat and white top-hat of the luminance image, which is shown in Figure 2c. It is clear that fewer of the lines have been extracted in this image than in Figure 2d, due to the colour information having been ignored in its calculation.

The version of this top-hat developed for angular images has been used to detect defects in textures which are characterised by having a dominant orientation different to that of the surroundings [3]. The angular information could be supplemented by a measure of angular coherence [6] or regularity [3], for example. Using the Euclidean distance to calculate the distance between these feature vectors allows their easy integration into the suggested top-hat function. An example demonstrating this application is shown in Figure 3, in which we attempt to use the information that the knots and other minor defects in wood perturb the surrounding vein orientations to locate the potential positions of defects. The angle and angular coherence images of Figure 3a are shown in Figures 3b and c respectively. They are calculated as described in [6], with a square neighbourhood of side length 32 pixels used for the angle images, and a square neighbourhood of side length 64 pixels used for the coherence image. Both neighbourhoods were moved in steps of size 16 pixels. The different neighbourhood sizes reflect the different information that we obtain from the features. The angles should be as constant as possible in a neighbourhood, as we later look for differences between the mean angular values calculated in adjoining neighbourhoods to locate defects. Low angular coherence is also an indicator of a defect being present, but low coherence is only obtained if the neighbourhood contains a large enough sampling of the region surrounding the defect.

The top-hat calculated for the angle image using Equation 4 with the distance D replaced by the acute angle between two angular valued pixels is shown in Figure 3d, and the standard greyscale top-hat applied to the coherence image is shown in Figure 3e. A square structuring element of side length 9 pixels was used. Higher pixel values in these top-hat images indicate a higher likelihood of a defect being present in the corresponding neighbourhood in the original image. The two small knots near the top are equally well detected by both top-hats. Only a part of the larger knot in the centre is detected by the coherence top-hat whereas the entire region is detected by the angular top-hat. Finally, the region of small white lines at the bottom right of the wood image is better detected by the coherence top-hat than by the angular top-hat. The suggested top-hat, which uses the Euclidean distance between the angle and coherence feature pairs, allows the detection ability of both features to be simultaneously taken into account, as shown in figure 3f.

4. Conclusion

There are three ways of expanding the morphological top-hat to multi-channel images:

1. Use the standard greylevel top-hat on a single channel.
2. Use a vectorial order for the opening or closing followed by a vectorial distance calculated between corresponding pixels in the initial and processed image to produce a greylevel image.
3. Rewrite the top-hat operator in terms of increments, which allows the direct substitution of a vectorial distance without having to choose a vectorial order.

The first approach is useful if it is known that all the information of interest is in one of the channels and it is not necessary to take into account the other channels when calculating the difference. The second approach allows all the channels to contribute to the distance, although when the lexicographical order is used, it is still required that the features of interest be encoded in the channel placed at the top level of the order. This approach would be useful, for example, in the CIELAB colour space where the Euclidean distance between the colour vectors has a physical interpretation. The third approach does not require an order to be chosen, only a distance function. Note that, in contrast to the standard white and black top-hats, it does not distinguish between lighter and darker pixels. It is sufficient that the vector values are different from the surrounding ones. Applications to feature detection in colour images and to defect detection in texture images have been presented, although the suggested approaches are applicable to any vectorial images.

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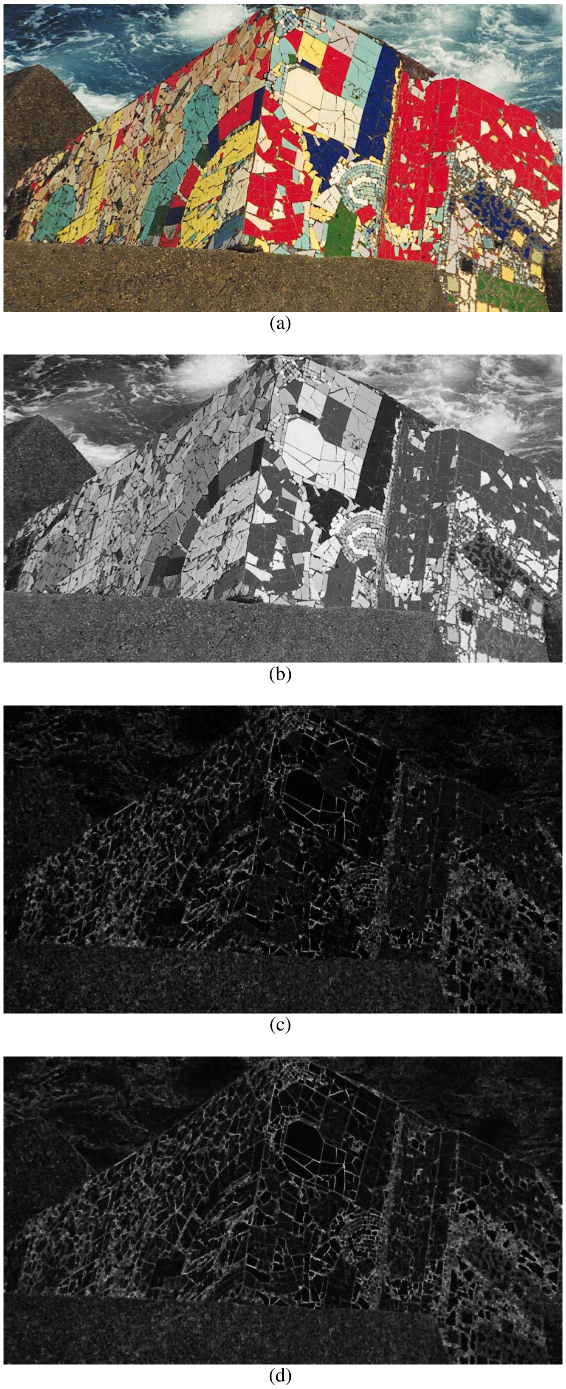

Figure 2. (a) Initial image (size 417x758 pixels). (b) Luminance channel. (c) Maximum of the white and black top-hats of the luminance image. (d) Proposed vectorial top-hat.

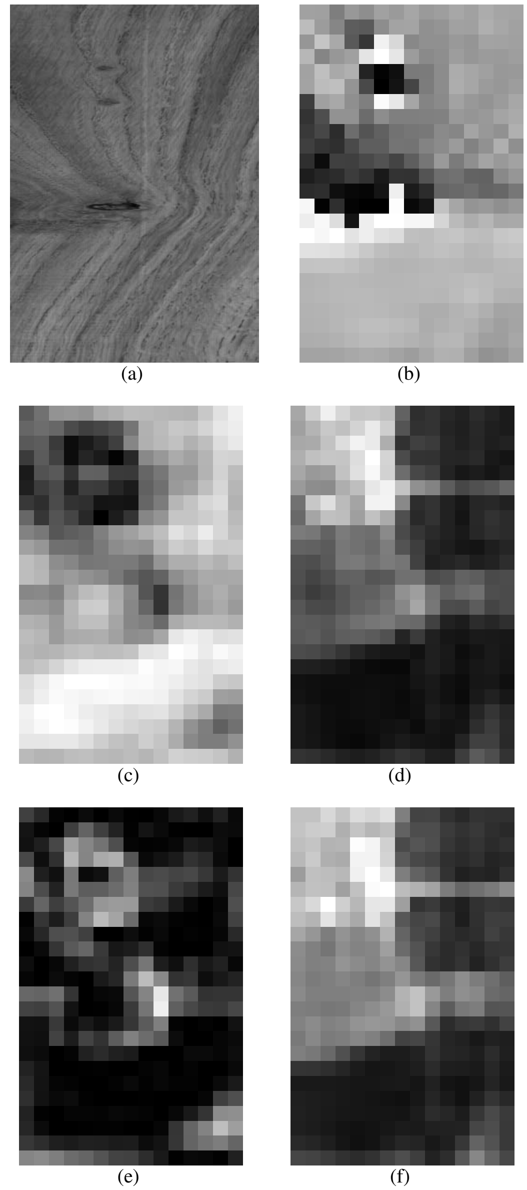

Figure 3. (a) Initial image (size 433x301 pixels). (b) Angle image. The grey-values encode the dominant vein direction in each neighbourhood, where a value of $90°$ represents vertical veins. (c) Coherence image. The grey-values encode the orientation coherence in each neighbourhood. (d) Angular top-hat of (b). (e) Black top-hat of (c). (f) Proposed combined top-hat. In the top-hat results, higher grey-values indicate a higher likelihood of a defect being present.

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