IHLS Transformations

Allan Hanbury

October 17, 2007

1 Transformations to and from the cylindrical coordinate space with $i = 1/3(R + G + B)$

1.1 RGB to cylindrical coordinates

$$i = \frac{1}{3}(R + G + B) \tag{1}$$

$$s = \max(R, G, B) - \min(R, G, B) \tag{2}$$

$$h = \arctan\left(\frac{\sqrt{3}(G - B)}{2R - G - B}\right) \tag{3}$$

1.2 Cylindrical coordinates to RGB

One first calculates the chroma values from the saturation values

$$c = \frac{\sqrt{3}s}{2\sin(120^\circ - H^*)} \tag{4}$$

where H^* is given by

$$H^* = H - k \times 60^\circ \text{where } k \in \{0, 1, 2, 3, 4, 5\} \text{ so that } 0^\circ \leq H^* \leq 60^\circ \tag{5}$$

From the chroma, one calculates

$$c_1 = c\,\cos(h) \quad \text{and} \quad c_2 = c\,\sin(h) \tag{6}$$

For the case where the hue is undefined: $c_1 = c_2 = 0$. To get the R, G and B values, the following are used:

$$R = i + \frac{2}{3}c_1 \tag{7}$$

$$G = i - \frac{1}{3}c_1 + \frac{1}{\sqrt{3}}c_2 \tag{8}$$

$$B = i - \frac{1}{3}c_1 - \frac{1}{\sqrt{3}}c_2 \tag{9}$$

2 Transformations to and from the cylindrical coordinate space with $i = 1/2[\max(R, G, B) + \min(R, G, B)]$

2.1 RGB to cylindrical coordinates

$$
\begin{aligned}
i &= \frac{1}{2}\left[\max\left(R, G, B\right) + \min\left(R, G, B\right)\right] && (10) \\
s &= \max\left(R, G, B\right) - \min\left(R, G, B\right) && (11) \\
h &= \arctan\left(\frac{\sqrt{3}(G - B)}{2R - G - B}\right) && (12)
\end{aligned}
$$

2.2 Cylindrical coordinates to RGB

One first calculates the chroma values from the saturation values

$$c = \frac{\sqrt{3}s}{2\sin\left(120^\circ - H^*\right)} \tag{13}$$

where H^* is given by

$$H^* = H - k \times 60^\circ \text{where } k \in \{0, 1, 2, 3, 4, 5\} \text{ so that } 0^\circ \leq H^* \leq 60^\circ \tag{14}$$

From the chroma, one calculates

$$c_1 = c\ \cos\left(h\right) \quad \text{and} \quad c_2 = c\ \sin\left(h\right) \tag{15}$$

For the case where the hue is undefined: $c_1 = c_2 = 0$ and $R = G = B = i$. Otherwise, the following are used:

If $k = 0$ or $k = 3$ (from Equation 14), then

$$
\begin{aligned}
R &= i + \frac{1}{2}c_1 + \frac{1}{2\sqrt{3}}c_2 && (16) \\
G &= i - \frac{1}{2}c_1 + \frac{\sqrt{3}}{2}c_2 && (17) \\
B &= i - \frac{1}{2}c_1 - \frac{1}{2\sqrt{3}}c_2 && (18)
\end{aligned}
$$

If $k = 1$ or $k = 4$, then

$$
\begin{aligned}
R &= i + c_1 && (19) \\
G &= i + \frac{\sqrt{3}}{3}c_2 && (20) \\
B &= i - \frac{\sqrt{3}}{3}c_2 && (21)
\end{aligned}
$$

If $k = 2$ or $k = 5$, then

$$R = i + \frac{1}{2}c_1 - \frac{1}{2\sqrt{3}}c_2 \tag{22}$$

$$G = i - \frac{1}{2}c_1 + \frac{1}{2\sqrt{3}}c_2 \tag{23}$$

$$B = i - \frac{1}{2}c_1 - \frac{\sqrt{3}}{2}c_2 \tag{24}$$